*PROBLEM 19: THE FREEZE-OUT OF A FICTITIOUS PARTICLE X

(25 points)

The following problem was Problem 3 of Quiz 3, 2016.

Suppose that, in addition to the particles that are known to exist, there also existed a family of three spin-1 particles, X^+, X^-, and X^0, all with masses 0.511 MeV/c², exactly the same as the electron. The X^- is the antiparticle of the X^+, and the X^0 is its own antiparticle. Since the X's are spin-1 particles with nonzero mass, each particle has three spin states.

The X's do not interact with neutrinos any more strongly than the electrons and positrons do, so when the X's freeze out, all of their energy and entropy are given to the photons, just like the electron-positron pairs.

(a) *(5 points)* In thermal equilibrium when $kT \gg 0.511$ MeV/c², what is the total energy density of the X^+, X^-, and X^0 particles?

(b) *(5 points)* In thermal equilibrium when $kT \gg 0.511$ MeV/c², what is the total number density of the X^+, X^-, and X^0 particles?

(c) *(10 points)* The X particles and the electron-positron pairs freeze out of the thermal equilibrium radiation at the same time, as kT decreases from values large compared to 0.511 MeV/c² to values that are small compared to it. If the X's, electron-positron pairs, photons, and neutrinos were all in thermal equilibrium before this freeze-out, what will be the ratio T_ν/T_γ, the ratio of the neutrino temperature to the photon temperature, after the freeze-out?

(d) *(5 points)* If the mass of the X's was, for example, 0.100 MeV/c², so that the electron-positron pairs froze out first, and then the X's froze out, would the final ratio T_ν/T_γ be higher, lower, or the same as the answer to part (c)? Explain your answer in a sentence or two.

(a)

$$u = g \frac{\pi^2}{30} \frac{(kT)^4}{(\hbar c)^3}$$

$$g \begin{cases} 1 & \text{per spin state for bosons} \\ 7/8 & \text{per '' '' '' fermions} \end{cases}$$

$$X^+, X^-, X^0 \quad g = \underset{\substack{\nearrow \\ \text{\# of species}}}{3} \times \underset{\substack{\uparrow \\ \text{spin states}}}{3} \times \underset{\substack{\nwarrow \\ \text{boson}}}{1} = 9 \quad \Longrightarrow \quad u = \frac{3\pi^2}{10} \frac{(kT)^4}{(\hbar c)^3}$$

$$\underset{\substack{\uparrow \\ 1}}{e^+} \underset{\substack{\uparrow \\ 1}}{e^-} : \; g = \underset{\substack{\nearrow \\ \text{species}}}{\boxed{2}} \times \underset{\substack{\uparrow \\ \text{spin states}}}{2} \times \underset{\substack{\nwarrow \\ \text{fermions}}}{\frac{7}{8}}$$

(b)

$$n = g^* \frac{\zeta(3)}{\pi^2} \frac{(kT)^3}{(\hbar c)^3}$$

$$g^* \begin{cases} 1 & \text{per spin state for bosons} \\ 3/4 & \text{'' '' for fermions} \end{cases}$$

$$X^+, X^-, X^0 \quad g^*_{tot} = \underset{\substack{\nearrow \\ \text{species}}}{3} \times \underset{\substack{\uparrow \\ \text{spin states}}}{3} \times \underset{\substack{\uparrow \\ \text{boson}}}{1} \quad \Longrightarrow \quad n = \frac{9\zeta(3)}{\pi^2} \frac{(kT)^3}{(\hbar c)^3} \;//$$

(c) After neutrinos decouple, but before X, e freeze out,

$$S_{tot} = S_\nu + S_{other} \Rightarrow a^3 s_\nu \text{ is constant}$$

separately conserved

$$a^3 s_{other} \text{ is constant}$$

In general, $s = A g T^3 \Rightarrow g_\nu \left(a_I^3 T^3\right)_{\substack{\text{After}\\\text{neutrinos}\\\text{decouple}}} = g_\nu \left(a_F^3 T_\nu^3\right)_{\substack{\text{any}\\\text{later}\\\text{time}}}$

$$\left(g_{other}^{\text{before freezeout}} a_I^3 T^3\right)_{\substack{\text{After}\\\text{neutrinos}\\\text{decouple}}} = \left(g_{other} a_F^3 T_\gamma^3\right)_{\substack{\text{After}\\\text{electrons, } X\\\text{freeze out}}} \qquad (*)$$

$$g_{other}^{\text{before freezeout}} = \overbrace{\underset{\text{species}}{2} \times \underset{\text{spin states}}{2} \times \underset{\text{fermions}}{7/8}}^{e^+e^-} + \overbrace{\underset{\text{species}}{1} \times \underset{\text{polarizations}}{2} \times \underset{\text{bosons}}{1}}^{\gamma} + \overbrace{\underset{\text{species}}{3} \times \underset{\text{spin states}}{3} \times \underset{\text{bosons}}{1}}^{X^{+/-,0}}$$

$$= 7/2 + 2 + 9 = 29/2$$

$$g_{other}^{\text{after freezeout}} = 2$$

$$(*) \Rightarrow \frac{29}{2} \underbrace{a_I^3 T^3}_{\substack{s_\nu \text{ conservation}\\ = a_F^3 T_\nu^3}} = 2 a_F^3 T_\gamma^3$$

$$\Rightarrow \frac{29}{2} \cancel{a_F^3} T_\nu^3 = 2 \cancel{a_F^3} T_\gamma^3 \Rightarrow T_\nu = \left(\frac{4}{29}\right)^{1/3} T_\gamma$$

(d) $\frac{T_\nu}{T_\gamma}$ is exactly the same as in part (c).

The entropy must be conserved from the initial state (with γ, e, X) to the final state (with only γ).